# Ferro fluid based squeeze film in porous annular plates considering the effect of transverse surface roughness

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**Abstract:** This article aims to analyze the behaviour of a ferrofluid squeeze film between transversely rough annular plates with the help of boundary conditions depending on the magnetization parameter. The stochastic averaging model of Christensen and Tonder for characterizing the surface roughness has been employed here. The related stochastically averaged Reynolds' type equation is solved to obtain the pressure distribution, leading to the derivation of load carrying capacity. The results presented reveal that a suitable boundary condition may help us in bringing down the adverse effect of roughness to a significant extent. But, the situation remains fairly better when negatively skewed roughness is in place. Besides, this type of bearing system supports certain amount of load in the absence of the flow, which does not happen in the case of conventional lubricant, based bearing system.

Key words: Annular plates, Squeeze film, Porosity, Magnetic fluid, Surface roughness, Pressure, Load carrying capacity.

### **1** Introduction

Wu [1] examined the effect of porosity on squeeze film behaviour in annular irrotational disks. The load carrying capacity was found to be reduced due to the porosity. Gupta and Vora [2] analysed the squeeze film behaviour between curved annular plates. The curvature was found to have considerable influence on the performance of the squeeze film. Lin et. al. [3] extended the configuration of [2] to discuss the Magneto hydrodynamic squeeze film characteristics between curved annular plates. Patel and Deheri [4] investigated the configuration of [2] by considering the lower plate as well as the upper plate along the surfaces generated by hyperbolic function. Subsequently, Patel and Deheri [5] modified the approach to consider both plates along the surfaces determined by secant functions. The investigations of [4], [5] confirmed that the magnetization had a significantly positive effect on the squeeze film performance in annular plates. Use of magnetic fluid as a lubricant modifying the performance of the bearing system has been very well recognised. Bhat and Deheri [6], [7] analyzed the performance of a magnetic fluid based squeeze film behaviour between curved annular disks and curved circular plates and found that the performance with the magnetic fluid as lubricant was relatively better than with a conventional lubricant.

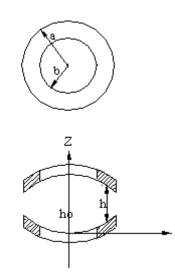
It is well known that bearing surfaces particularly, after having some run in and wear develop roughness. Various methods have been proposed to study and analyze the effect of surface roughness of the bearing surfaces on the performance of squeeze film bearings. Several investigators have adopted a stochastic approach to mathematically model the randomness of the roughness. A comprehensive general analysis was presented by Christensen and Tonder [8], [9], [10] for surface roughness (both transverse as well as longitudinal) based on a general probability density function. Later on, this approach of [8], [9], [10] laid down the basis of the analysis to discuss the effect of surface roughness on the performance of the bearing system in a number of investigations. Ting [11] discussed the engagement behaviour of lubricated porous annular disks by considering the effect of surface roughness on the squeeze film. The roughness significantly affected the performance characteristics. Gupta and Deheri [12] studied the effect of transverse surface roughness on the squeeze film performance in a spherical bearing. The transverse surface roughness was found to bring down the load carrying capacity but the situation was fairly better in the case of negatively skewed roughness when moderate to large values of variance(-ve) was involved. Vadher et. al. [13] investigated the performance of a hydromagnetic squeeze film between two conducting rough porous annular plates. The hydromagnetization resulted in a relatively better performance for all values of the conductivity parameter.

Deheri et. al. [14] dealt with the comparative study of magnetic fluid lubrication of squeeze films between rough curved annular plates. This study provided an additional degree of freedom from design point of view. Shimpi and Deheri [15] studied the performance characteristics of a ferrofluid based squeeze film in curved porous rotating rough annular plates considering the effect of deformation. Here, the deformation induced adverse effect was minimised by the ferrofluid lubrication in the case of negatively skewed roughness. Recently, Shimpi and Deheri [16] modified the above procedure to overcome the combined adverse effect of deformation, standard deviation associated with roughness and porosity.

The above studies neglected the effect of magnetization while forming the boundary conditions. In the present article it has been thought proper to make use of the effect of magnetization on the boundary conditions for a squeeze film performance in rough porous annular plates under the presence of a ferrofluid.

### 2 Analysis

The configuration of the bearing system presented below consists of the annular disks.



### Figure I: Geometry of the bearing system

The upper face moves normal towards the lower disk with uniform velocity  $\dot{h} = \frac{dh}{dt}$ . Both the disks are considered to have transversely rough surfaces. Assuming an axially symmetric flow of

the magnetic fluid lubricant between the disks under an oblique magnetic field  $\overline{H}$  whose magnitude H is a function of r vanishing at r = a (outer radius) and r = b (inner radius), the modified Reynolds' equation governing the film pressure ([17], [15]) is obtained as

$$\frac{1}{r}\frac{d}{dr}\left[Ar\frac{d}{dr}\left(p\mu 0\mu H_{0}\right)\right]^{2}=12\mu h \qquad \dots (1)$$

where

$$H^2 = (a - r)(r - b); b < r < a$$

 $A = h^3 \alpha 3h^2 h\alpha + 3h^2 \sigma + \varepsilon^2 3\sigma \alpha + \alpha^2 + 12^3 H \phi$   $\mu_0$  is permeability of the free space,  $\overline{\mu}$  is the magnetic permeability,  $\mu$  is the viscosity of the fluid,  $\sigma$  is standard deviation,  $\alpha$  is the variance and  $\varepsilon$  is the measure of skewness,  $\phi$  is permeability of porous facing. The boundary conditions

$$P(1)=0$$

and

$$\left(\frac{d\overline{P}\mu}{dR}\right)_{R=k_1} = -\frac{*}{2} \qquad \dots (2)$$

The dimensionless form of the equation (1) is

$$\frac{1}{R}\frac{d\mu}{dR}\left[\frac{-d}{AR\pi(1)}\left\{k^{-}\right\} + \frac{2}{1}\left(k\frac{-}{2}R\right)\left(R-1\right) = 1\right] \\ \frac{1}{R}\left[\frac{d\mu}{dR}\left\{k^{-}\right\} + \frac{2}{1}\left(k\frac{-}{2}R\right)\left(R-1\right) = 1\right] \\ \dots (3)$$

where

$$\sigma^{*} = \frac{\sigma}{h}, \qquad \alpha^{*} = \frac{\alpha}{h}, \qquad \varepsilon^{*} = \frac{\varepsilon}{h^{3}},$$
$$\psi = \frac{\phi H}{h^{3}}, \qquad k_{1} = \frac{a}{b}, \qquad R = \frac{r}{b}$$
$$\overline{A} = 1 + \sigma_{3} \left( \begin{array}{c} * \\ + \right)^{2} \alpha \end{array} + \begin{array}{c} * \\ 3 \alpha \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \sigma \end{array} \left( \begin{array}{c} * \\ \alpha \end{array} \right)^{2} + \begin{array}{c} * \\ \alpha \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \varepsilon \end{array} + \begin{array}{c} 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \sigma \end{array} \left( \begin{array}{c} * \\ \alpha \end{array} \right)^{2} + \begin{array}{c} * \\ \alpha \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \varepsilon \end{array} + \begin{array}{c} 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \sigma \end{array} \right)^{2} + \begin{array}{c} * \\ \alpha \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \varepsilon \end{array} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \varepsilon \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \left( \begin{array}{c} * \\ + \right)^{2} \varepsilon \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} = \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} = \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} \right)^{2} + \begin{array}{c} * \\ 1 + \sigma_{3} \phi \end{array} = \begin{array}{c} * \\ 1 + \sigma_{3} \phi$$

$$\mu^{*} = -\frac{h_{\mu}^{3}}{\mu h} , \quad \overline{P} = -\frac{h^{3}p}{\pi \mu h(a^{2} - b^{2})},$$

$$\overline{w} = -\frac{h^3 w}{\pi \mu \dot{h} (a^2 - b^2)^2}$$

Integrating the above equation with respect to the

boundary conditions (2) in dimensionless form pressure P is

$$\overline{P} = \frac{\mu^{*}(k_{1} - R)(R - 1)}{2\pi(k_{1}^{2} - 1)} + \frac{3(k_{1}^{2} - R^{2})}{\pi\overline{A}(k_{1}^{2} - 1)} + \ln\left(\frac{R\mu}{k_{1}}\right)\left[-\frac{*}{k_{2}^{2} + 12}\frac{\mu}{(1 - 1)} + \frac{6}{\pi\overline{A}(k_{1}^{2} - 1)}\right] \dots (4)$$

The load carrying capacity of the bearing is given by

$$\overline{w} = 2 \frac{k_1}{1} P dR$$

$$= \mu^* \left\{ \frac{k_1^2 + k_1 - 2}{12} + \frac{\pi}{4} \left( k_1^2 - 1 \right) \cdot \ln k_1 \left[ \pi + \frac{1}{k_1 + 1} \right] \right\}$$

$$+ \frac{3}{A} \left[ \frac{k_1^2 - 3}{2} + \frac{2}{k_1^2 - 1} \ln k_1 \right] \dots (5)$$

The response time  $\Delta t$  taken by the upper plate to reach a film thickness  $h_2$  startup from an initial film thickness  $h_1$ , can be determined in dimensionless form, as

$$\Delta t = -\frac{\overline{h_2}}{w \int_{h_1}^{h_2} \frac{d\overline{h}}{g_1(\overline{h})}}$$

where

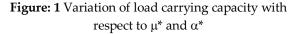
$$g_{1}(\bar{h}) = \bar{h}^{-3} + c\beta \left( \begin{array}{c} * \\ h \end{array} \right)^{2} + \bar{3}h \ \alpha^{-2} * \\ + \bar{3}h \left( \begin{array}{c} * \\ h \end{array} \right)^{2} + \bar{3} \ \left( \begin{array}{c} * \\ \end{array} \right)^{2} + \bar{3} \ \left( \begin{array}{c} * \\ \end{array} \right)^{2} \alpha + \left( \begin{array}{c} * \\ \end{array} \right)^{3} + \varepsilon + 12\psi \\ \bar{h} = \frac{h}{h_{0}}, \qquad \bar{h}_{1} = \frac{h_{1}}{h_{0}}, \qquad \bar{h}_{2} = \frac{h_{2}}{h_{0}} \end{cases}$$

### **3 Results and Discussions**

It is manifest in (5) that the increase in load carrying capacity due to the magnetization turns out to be

$$\mu^{*}\left\{\frac{k_{1}^{2}+k_{1}-2}{12}+\frac{\pi}{4}\left(k_{1}^{2}-1\right)-\ln k_{1}\left[\pi+\frac{1}{k_{1}+1}\right]\right\}$$

in comparison with the conventional lubricant based bearing system.



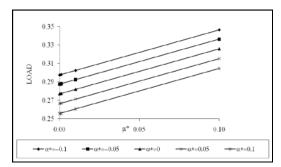
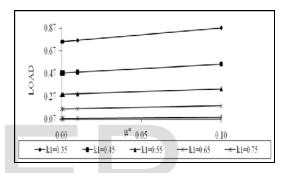


Figure: 2 Variation of load carrying capacity with respect to  $\mu^*$  and  $k_1$ 



The results presented in graphical forms (Figures (1)-(2)) suggest that the load carrying capacity increases sharply as the magnetization parameter increases.

Figure: 3 Variation of load carrying capacity with respect to  $\sigma^*$  and  $\epsilon^*$ 

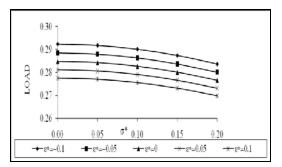
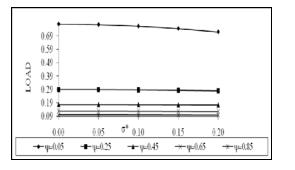


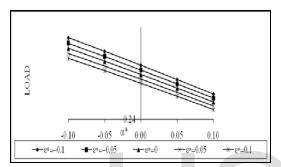
Figure (3) indicates the extent in mitigating the adverse effect of standard deviation in the case of negatively skewed roughness.

Figure: 4 Variation of load carrying capacity with respect to  $\sigma^*$  and  $\psi$ 



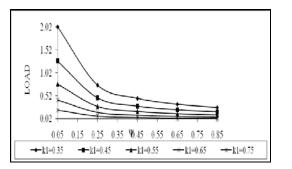
The fact that as the combined effect of porosity and standard deviation pulls down the load carrying capacity as can be seen from Figure (4).

Figure: 5 Variation of load carrying capacity with respect to  $\alpha^*$  and  $\epsilon^*$ 



It is appealing to note from Figure (5) that the increased load due to variance (-ve) further increases owing to the negatively skewed roughness.

Figure: 6 Variation of load carrying capacity with respect to  $\psi$  and  $k_1$ 



The decrease in the load carrying capacity due to porosity gets augmented when smaller values of aspect ratio is involved which is suggested by Figure (6).

# **4** Conclusions

From bearing's life period point of view this investigation suggests that the roughness aspect needs to be evaluated while designing the bearing system even if, magnetization parameter finds a place in the boundary conditions and suitable magnetic strength is in place.

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